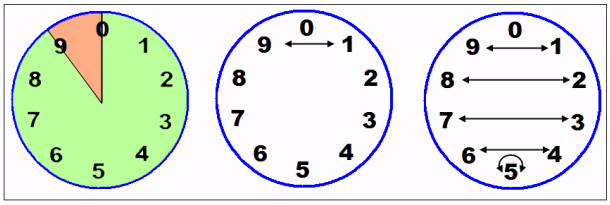
Overview

This is a quick overview of everything in Stage I
for those of you who are in a hurry and have the
mathematical background to take it all in.
Don't forget if you are a teacher there are free
teaching games, notes and exercises
which might be useful for you
on the Stage I page.

The "Add a Complement" Method

In a 10-circle below 9 and 1 are complementary opposites.
Together they add to one complete 10-circle.



We say 9 and 1 are "complements" of one another.
Complements add to 10 in a 10-circle.

To get the last digit of a subtraction
when it goes "below zero" and is "hard":

Add the complement
instead.

Example: 34 - 9 = 25

4 - 9 goes "below zero" so
the power digit goes down into the twenties
The answer is "20-something"...
The last digit of the answer to 34 - 9 is:
4 + 1 = 5 (add the complement of 9 to the 4)
The full answer is 34 - 9 = 25.

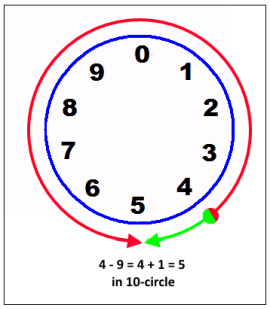
We only do this if the end digits subtract below zero.
43 - 2 = 41 just as one would expect.
Using the method here would result in:
43 - 2 = 3 '11' which is an error, but note 30 + 11 = 41.

The reason the method works can be explained two ways:

$$\begin{aligned} 34 - 9 &= 34 - 10 + 1 \\ &= (34 - 10) + 1 \\ &= 24 + 1 \\ &= 25 \end{aligned}$$

OR

by noting 4 - 9 = 4 + 1 = 5 around a 10-circle:



Learning to think in circles will prove valuable.

Repeat Subtraction

The "Add a Complement" strategy is particularly good for repeat subtraction because you either do an easy subtraction

or

you add the same complement each time:

9 5
<u>- 9</u>
8 6 ;5 - 9 won't go so use 5 + 1 = 6 for the last digit
<u>- 9</u>
7 7 ;6 - 9 becomes 6 + 1 = 7
<u>- 9</u>
6 8 ;7 - 9 becomes 7 + 1 = 8
<u>- 9</u>
5 9 ;8 - 9 becomes 8 + 1 = 9
<u>- 9</u>
5 0 ;59 - 9 = 50 is straightforward
<u>- 9</u>
4 1 ;0 - 9 becomes 0 + 1 = 1

Don't bother writing out the "-9" again and again.
Just write the answers under one another:

95
86
77
68
59
50
41

It is quicker, easier and clearer to do so.

Notice how easy this strategy makes subtraction with just a little practice.
To subtract 8's repeatedly just add 2 when it won't go.
Add 3 for subtraction by 7...
And so on.

It's time to

Before continuing it is vital to actually practice this strategy
so you can appreciate just how easy repeat subtraction can be.
If you don't grasp that you will miss the point
of what follows entirely.

Writing out the Times Tables

Start with 90 and take away 9 repeatedly.
Start with 80 and take away 8 repeatedly.
Start with 70 and take away 7 repeatedly.

You have written out the 7, 8 and 9 times tables!

10 x 9 = 90
So if you take away one lot of 9 then it follows
9 x 9 = 81
and take away another lot of 9 giving:
8 x 9 = 72
and so on.

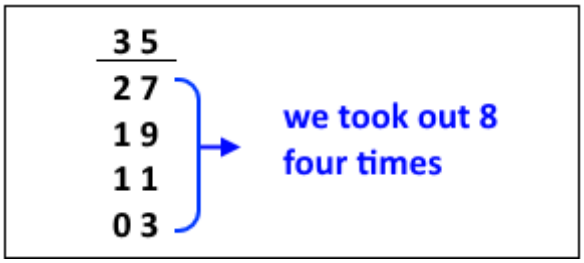
So anyone who can't remember the tables but who can
subtract fast can write out the times tables in moments.

Long Division

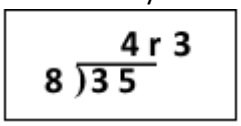
It is presumed at this point that if you were given
any starting number (such as 35)
that you would be able to repeatedly subtract
all the 8's out of it (for example)
with little to no effort as quick as a flash:

3 5
2 7
1 9
1 1
0 3

We can't take any more 8's out of 03 so we stop.
How many 8's were in 35?
By count there must have been 4 lots of 8:



That is what we mean by "dividing" 35 by 8:

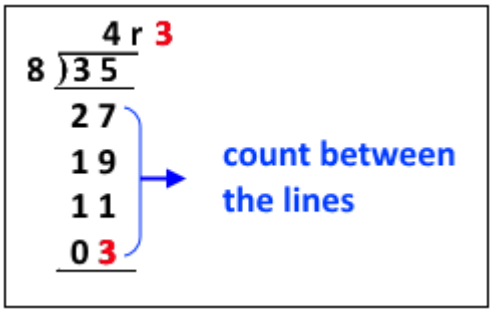


We say "How many 8's in 35".
By subtracting we actually took the 8's out
so we can count up how many were in 35.

In the subtraction we stopped when we reached "03".
We couldn't take another 8 out.
That is the remainder.

Repeat subtraction
is division
just as
repeat addition
is multiplication.

So even a 6 year old child could learn to
do long division by repeat subtraction:



Tip: Draw underlining where shown.
This helps when counting up the answer.
The remainder is the number left over
which can't be divided.

Benefits:

The benefits of introducing division like this to a child are:

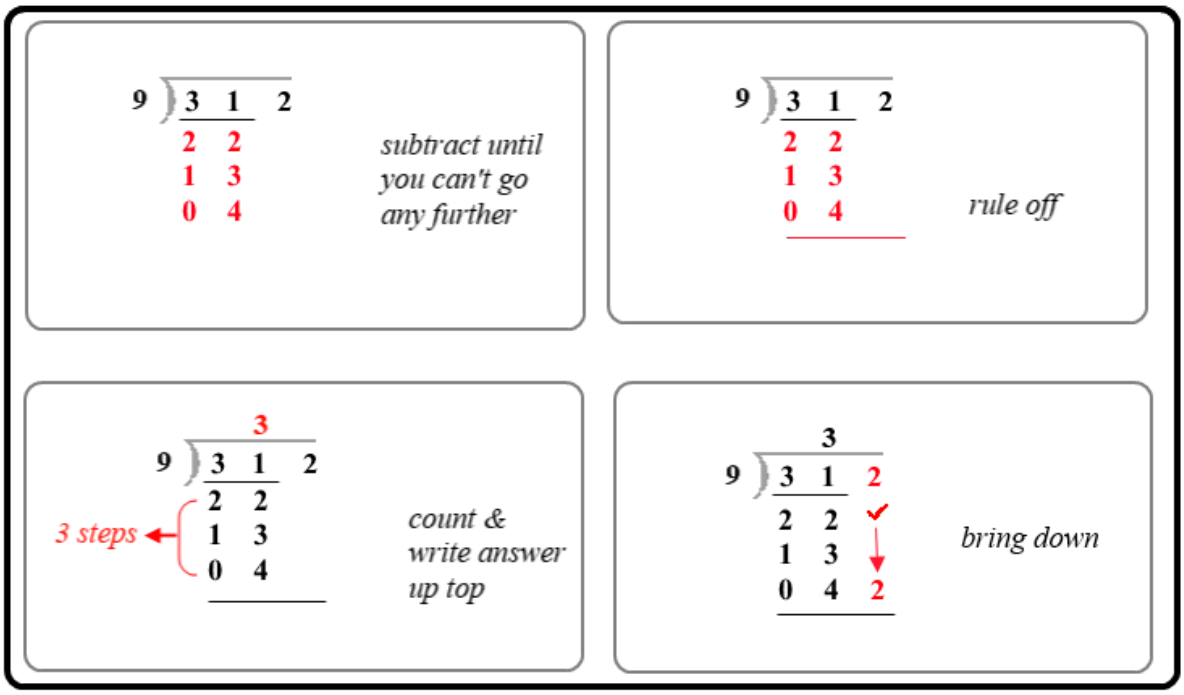
1. You can introduce the concept earlier because repeat subtraction is literally child's play and doesn't require memorization of the times tables.

2. Division is repeat subtraction so this is dealing most directly with division, instead of relying on an abstract and complicated algorithm which might be beyond the child's grasp.

3. This can easily be bridged to more formal algorithms if and when one wished.

Long Division with "Bringing Down"

Here is an example where we bring the next digit down
just as in the standard way of doing long division:



This pretty much covers the essential idea.
The other articles and videos go into more detail.